Math Diversion Problem 609

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Dear Algebra, stop asking us to find your X,
she's not coming back.
— Woody Paige

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=UyTPznZpLSg
Title: Definite Integral x(x^2 + 1)^3 from 0 to 1
 Presenter: Cole's World of Mathematics
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1 The Problem

Integrate the following integral:

$$I = \int_0^1 x(x^2+1)^3 dx\,. \tag{1}$$

2 The Solution

$$\begin{aligned} I &= \int_0^1 x(x^2+1)^3 dx && \text{(2a)}\\ &= \frac{1}{2}\int_0^1 (x^2+1)^3 d(x^2+1) && \text{(2b)}\\ &= \frac{1}{8}(x^2+1)^4\Big|_0^1 && \text{(2c)}\\ &= \frac{1}{8}\left[(1^2+1)^4 - (0^2+1)^4\right] && \text{(2d)}\\ &= \frac{1}{8}[\,16-1\,] && \text{(2e)}\\ &= \frac{15}{8}\,. && \text{(2f)} \end{aligned}$$