Math Diversion Problem 616

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> You either get control of your lusts and feelings of entitlement, or they will get control of you.
> — The Author

The YouTube video is found at:

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Source: Many locations
Title: Final-ratio problem
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1 The Problem

A can contains a mixture of two liquids A and B in ratio $7 : 5$. After 9 liters are drawn off and replaced by 9 liters of liquid B, the ratio of A to B becomes $7 : 9$. How many liters of liquid A was in the can initially.

2 Solution

Let x represent the original total liquid contents of the can. Since we draw off 9 liters and replace it by 9 liters, the final liquid will have x liters in it. Once we determine x, we can then solve for the initial value of A in the can. To analyze the figure, our 'effective' starting condition the state just after the 9 liters of fluid has been drawn off the can. (But this is not the actual initial state.)

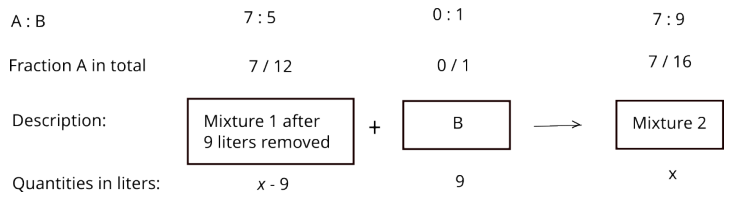

Figure 1. This graphic represents the adding some water x to a starting liquid in a 'before and after' process. 'Fraction A in total' means A/(A+B).

We begin with our usual conservation equation, this time for A.

$$(\text{total } A \text{ in can before adding } B) = (\text{total } A \text{ in can after adding } B)\,. \tag{1}$$

Putting in some details, we get

$$\frac{7}{12}(x-9) + 0 \cdot 9 = \frac{7}{16}(x)\,, \tag{2}$$

or

$$4(x-9) = 3x\,, \tag{3}$$

which gives us

$$x = 36\,. \tag{4}$$

Now, this is the total liters of mixture in the can before the 9 liters has been removed. Also, the ratio of A to B did not change because some of it was removed. Therefore I interpret this as the initial state of the fluids in the can as having the ratio of $7:5$ for the ratio of A to B at the start. Hence, we set up the conversion factor as $\frac{7}{12}$:

$$\text{Initial liters of } A \text{ in can} = \frac{7}{12} \times 36 = 21\,. \tag{5}$$

Therefore, the initial liters of A in can is 21 L.