Math Diversion Problem 617

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I love it when a plan comes together.
— Hannibal Smith, *The A-Team*

1 Word Problem: The Boron Problem

▶ Naturally occurring Boron (molar mass of 10.81 g/mol = $10.81\,\mathrm{g{\cdot}mol^{-1}}$) is the mixture of two of its isotopes: Boron 10 (^{10}B) and Boron 11 (^{11}B), of atomic masses 10.01 g/mol and 11.01 g/mol, respectively. Find the relative abundances of ^{10}B and ^{11}B in natural Boron, expressed in percentages.

SOLUTION:

Conceptualizing the problem

First, take note that the mixing of these two isotopes to form natural Boron is a physical, not a chemical, mixing. We shall model this problem as mixing the two isotopes in the right proportions to yield naturally occurring Boron. We lose nothing by assuming one mole of natural boron to begin with.[1]

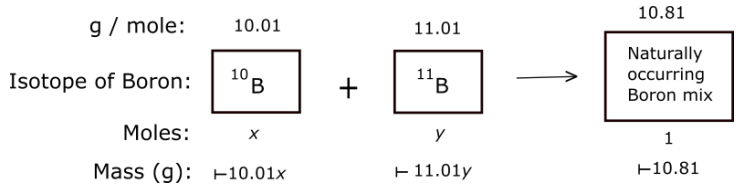

Figure 1. This graphic represents our imagined sorting of 1 mole of naturally occurring Boron into logical piles of ^{10}B and ^{11}B.

In this problem, we have one equation for the conservation of the total number of moles of isotopes, and another for the conservation of the total grams of

[1] If this were not the case, then the relative percentages of these two isotopes of boron in natural boron would be functions of the macroscopic amount of natural boron collected, all other things being equal. But if this were true, then claiming that natural boron has the fixed molar mass of 10.81 would be untrue and/or meaningless.

the isotopes, from which we get the pair of equations to be solved simultaneously:

$$x + y = 1\,, \tag{1a}$$

$$10.01x + 11.01y = 10.81\,. \tag{1b}$$

Getting the Numbers.

Wolframalpha gives the approximate values for x and y as $x \approx 0.2$ and $y \approx 0.8$. Converting these values to percentages, we have that in naturally occurring Boron, the relative abundance of ^{10}B as about 20% and ^{11}B as about 80%.