Math Diversion Problem 620

P. Reany

May 31, 2025

> After a time, you may find that having is not so
> pleasing a thing after all as wanting. It is not
> logical, but is often true.
> — Spock, Star Trek TOS

The YouTube video is found at:

```
Source: The Ether of Mathematical Ideas
Title: Ratios vs Fractional Amounts
 Presenter: Patrick
```

1 Problem

A composite fluid is composed of precisely two distinct fluids A and B. Given the ratio of $A : B = 4 : 3$, show that the fractional conversion factor amounts of A and B in the composite are

$$\frac{4}{7} \quad \text{and} \quad \frac{3}{7}\,, \tag{1}$$

respectively.

2 Solution

It stands to reason that the forms of these fractions are

$$\frac{4}{x} \quad \text{and} \quad \frac{3}{x}\,, \tag{2}$$

for A and B, respectively, and x is to be determined. This is obvious if the amount of A and B in the composite is to have the correct given ratio:

$$\frac{4/x}{3/x} = \frac{4}{3} \sim 4 : 3\,. \tag{3}$$

Now, let $\overline{A}$ and $\overline{B}$ represent the actual amounts of A and B in the composite, respectively, and $\overline{C}$ is their sum. Then

$$\overline{A} + \overline{B} = \overline{C}\,, \tag{4}$$

Next, we replace $\bar{A}$ and $\bar{B}$ by their calculated amounts, to get

$$\frac{4}{x}\,\bar{C} + \frac{3}{x}\,\bar{C} = \bar{C}\,. \tag{5}$$

On multiplying through by $\dfrac{x}{\bar{C}}$, we get

$$4 + 3 = x\,, \tag{6}$$

and therefore,

$$x = 7\,, \tag{7}$$

which is what we needed to show.

3 Conclusion

This problem is the key to how all similar fractional amounts relate to a given ratio, in similar circumstances.

So, if if the ratio of $\bar{A}$ and $\bar{B}$ is $a : b$, say, then their respective fractional conversion factors are,

$$\frac{a}{a+b} \quad \text{and} \quad \frac{b}{a+b}\,. \tag{8}$$