# Math Diversion Problem 621

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> To guard against a problem [occurring], you need
> first to imagine that it can happen.
> — Dave Farley

The YouTube video is found at:

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Source: The Ether of Mathematical Ideas
Title: Balancing chemical equation with linear algebra
 Presenter: Patrick
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## 1 Introduction

QUESTION: Can we use algebra to balance chemical equations? ANSWER: Often!

We're going to use the methods of algebra to solve for the coefficients to unbalanced chemical equations. For some reactions this is not possible as there are more than one possible combination of coefficients that will solve the equations. But for the given reaction below, the coefficients are unique up to an overall multiplicative factor. In this case, we can use linear algebra to solve for the proper coefficients. However, if the coefficients are not unique, one has to use the standard methods, such as redox.

## 2 Problem

Using algebraic methods, balance the following unbalance chemical equation:

$$\mathrm{KCN} + \mathrm{FeCl_2} \rightarrow \mathrm{K_4Fe(CN)_6} + \mathrm{KCl}\,. \tag{1}$$

Goal Statement: Find the coefficients that balance all the elements on the equation.

## 3 Solution

Now, we could provide each term its own coefficient to solve for. such as, So, (1) becomes

$$x_1\text{KCN} + x_2\text{FeCl}_2 = x_3\text{K}_4\text{Fe(CN)}_6 + x_4\text{KCl}\,. \tag{2}$$

However, we only need to solve for three unknowns since the coefficients are only determined up to their mutual ratios.

Therefore, we can simplify the problem by setting any one of the coefficients to unity, Say we choose $x_1$ for that. Thus (2) becomes

$$x_1\text{KCN} + x_2\text{FeCl}_2 = x_3\text{K}_4\text{Fe(CN)}_6 + \text{KCl}\,. \tag{3}$$

Now, we can think of this as a conservational problem: The total number of each element is conserved going from the left-hand side to the right-hand side.[1] Every total is equal to the sum of its parts. What are the parts, then? The parts are the contributions of the particular element from each term. Therefore we can write the conservational equation for Potassium, K, yielding

$$\text{Total K on LHS} = \text{Total K on RHS} \tag{4}$$

and the two others we need follow similarly. Then summing up and equating the term-wise contributions for K, N, and Fe gives

$$\begin{array}{lcccccccc} \text{K} & : & 1x_1 & + & 0x_2 & = & 4x_3 & + & 1 \\ \text{N} & : & 1x_1 & + & 0x_2 & = & 6x_3 & + & 0 \\ \text{Fe} & : & 0x_1 & + & 1x_2 & = & 1x_3 & + & 0 \end{array} \tag{5}$$

There are a number of ways to solve (5). One way is to subtract the first equation from the second, yielding $x_3 = \frac{1}{2}$. Substituting this value into the second gives $x_1 = 3$. And from the third equation we get $x_2 = x_3 = \frac{1}{2}$. Thus (3) becomes

$$3\text{KCN} + \tfrac{1}{2}\text{FeCl}_2 = \tfrac{1}{2}\text{K}_4\text{Fe(CN)}_6 + \text{KCl}\,. \tag{6}$$

On multiplying this through by 2 we get

$$6\text{KCN} + \text{FeCl}_2 = \text{K}_4\text{Fe(CN)}_6 + 2\text{KCl}\,. \tag{7}$$

We got the coefficients for K, N, and Fe, sure enough, but we're not finished yet. According to the goal statement, we still need to verify that the coefficients work for C and Cl, which they do, and this can easily be proved by inspection.

[1] In saying 'number', we can think in terms of individual atoms or in terms of moles.