Math Diversion Problem 622

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> What mathematicians are really interested in are
> coming up with interesting theorems and proofs.
> — Timothy Nguyen

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=qP0mghCvG64
Title: Italy l can you solve??
 Presenter: Math Master TV
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1 The Problem

Given the relation

$$x^{27} = 27^{x^2}\,, \tag{1}$$

find the real values of x.

2 The Solution

Let's begin with an α substitution:

$$x = 27^{\alpha}\,. \tag{2}$$

(Calculations continue after the table!)

α	27α	$27^{2\alpha}$
0	0	1
1	27	27^2
1/3	9	9 ✓

Table 1: Alpha is a fraction this time!

Substituting this into (1), we get

$$27^{27\alpha} = 27^{27^{2\alpha}}\,. \tag{3}$$

After equating exponents, we have that

$$27\alpha = 27^{2\alpha}\,. \tag{4}$$

Now, let's use a table! Therefore,

$$x = 27^{1/3} = 3\,. \tag{5}$$

WolframAlpha tells me that there's another real value of about 1.18802.