Math Diversion Problem 626

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> Linear Algebra is about the relation between
> the columns and the rows.
> — Gilbert Strang

The YouTube video is found at:

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Source: ??
Title: Mixed-rate problem
 Presenter: Patrick
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1 The Problem

Consider the following problem: Printer #1 can print 100 copies of a document in 3.4 hours and Printer #2 can print out the same print job in 2.5 hours. How long will it take for the print job to complete if both printers work on the job together, starting and stopping at the same times?

Note: It's quite likely in problems like this that the answers will be only approximates.

2 Solution: getting the equations to solve

We introduce the shorthand 'part of job done by' → PJDB. Then our highest-level equation is[1]

$$1 \text{ job} = (\text{PJDB Printer 1}) + (\text{PJDB Printer 2})\,. \tag{1}$$

Let R_1 be the average rate at which Printer 1 can work, which is 1 job/3.4 hours. Likewise, R_2 is the average rate at which Printer 2 can work, which is 1 job/2.5 hours. Now, the most general expression we can write for the refinement of the

[1] We're using a top-down analysis, in which we first state the simplest equation we can find, framing it in mostly just natural language. Later on, we can perform a stepwise refinement to end up with a conventional algebraic equal. Don't try to accomplish this all at once unless your name is Terence Tao.

last equation is[2]

$$1 \text{ job} = R_1 T_1 + R_2 T_2 \,, \tag{2}$$

where T_1 and T_2 are the respective times that Printer 1 and Printer 2 are operating. For the current problem, these two times are equal and they are equal to the total time the print job takes, but the last equation is the most general for two printers. So, for our current problem, let's set this common time equal to T and suppress units, to get

$$1 = R_1 T + R_2 T = (R_1 + R_2) T \,. \tag{3}$$

Perhaps this equation is beginning to look familiar to you from your previous algebra experience (such as in the problems given in the SAT, GRE, or LSAT). Solving for T, we get

$$T = \frac{1}{R_1 + R_2} = \frac{1}{\dfrac{1}{3.4} + \dfrac{1}{2.5}} \,. \tag{4}$$

3 Solution: getting the numbers

Anyway, using the given values of R_1 and R_2 plugged into (4), we get $T = 1.44$ hours $\approx$ 1 hour 26 minutes.

But does this answer make sense? Well, it's less time for them working together than for the faster printer to accomplish the job alone. That makes sense.

4 Afterthoughts

But that leaves us to consider why we were told that the print job constituted 100 copies. I point this out because we never used that given information in the answer. The short answer is that sometimes we are given more information than is actually needed to solve the problem (and that can be confusing). On the other hand, if we had been asked how many of the 100 copies had been printed by Printer 2, then that information would have been relevant.

So, let's make it relevant! How many were printed by Printer 2?

$$\begin{aligned} \#\text{ printed by Printer 2} = \text{Printer 2's rate} \times T &= \frac{100 \text{ copies}}{2.5 \text{ hr}} \times 1.44 \text{ hr} \\ &\approx 57.6 \text{ copies} \,. \end{aligned} \tag{5}$$

And this is what I meant by the answers being often approximations. If the manager of the printer job knew these results going in, he or she might well have Printer 2 run off 58 copies, and leave the rest for Printer 1.

[2] We are employing the Zeroth Rule of Problem Solving to make the simplifying assumption that the average rate will be accurate for arbitrarily long or short time intervals.