Math Diversion Problem 627

P. Reany

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> Physical concepts are free creations of the human mind, and
> are not, however it may seem, uniquely determined
> by the external world.
> — Albert Einstein
> (similar to scientific instrumentalism)

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=b2FV-LMoqcE
Title: An Exponent That Conjugates | Problem 533
 Presenter: aplusbi
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1 Problem

Given the relation

$$(1-i)^z = 1+i\,, \tag{1}$$

find the complex values of z.

2 Solution

If we multiply (1) through by $1-i$, we get

$$(1-i)^{z+1} = (1+i)(1-i) = 2\,, \tag{2}$$

Then, of course, we take the natural logarithm:

$$z+1 = \frac{\ln 2 + 2\pi i n}{\ln(1-i)} \qquad n \in \mathbb{Z}\,, \tag{3}$$

But

$$1-i = \sqrt{2}e^{-i\pi/4}\,. \tag{4}$$

Therefore,

$$z = \frac{\ln 2 + 2\pi i n}{\frac{1}{2}\ln 2 - i\pi/4} - 1\,. \tag{5}$$