Math Diversion Problem 640

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> Creation is an act of shear will.
> — John Hammond
> 'Jurassic Park'

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=iiSQZ5dU8oA
Title: Can you Solve Stanford University Admission Interview Exam?
 Presenter: Super Academy
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1 The Problem

Given the relation

$$729^x - 9^x = 2\sqrt{3}\,, \tag{1}$$

find the real values of x.

2 The Solution

The number 729 is special: $729 = 9^3$. Thus (1) becomes:

$$9^{3x} - 9^x - 2\sqrt{3} = 0\,. \tag{2}$$

Now, since I have an aversion to the $\sqrt{3}$, let's make the variable substitution:

$$9^x = \sqrt{3}y\,, \tag{3}$$

and thus (2) becomes

$$3y^3 - y - 2 = 0\,. \tag{4}$$

Now this looks like something I can deal with. On trying some small integer values for y, I found $y = 1$ works. That leaves me with the partially factored cubic:

$$(y-1)(3y^2 + 3y + 2) = 0\,. \tag{5}$$

However, $3y^2 + 3y + 2 = 0$ has no real roots, which leaves us with just $y = 1$ to use in (3). Therefore

$$9^x = \sqrt{3}\,, \tag{6}$$

from which we get

$$x = 1/4\,. \tag{7}$$