Math Diversion Problem 644

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> Finally, brethren, whatsoever things are true, whatsoever things are honest, whatsoever things are just, whatsoever things are pure, whatsoever things are lovely, whatsoever things are of good report; if there be any virtue, and if there be any praise, think on these things.
> — A Bible Verse for a healthier mindset

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=GRU3MhyxO6A
Title: Harvard University Admission Interview Tricks
 Presenter: Super Academy
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1 The Problem

Given the relations

$$3^x - 3^y = 16\,, \tag{1}$$

$$3^{x+y} = 4\,, \tag{2}$$

find the values of x and y.

2 The Solution

Let's begin by multiplying (1) through by 3^y:

$$3^{x+y} - 3^{2y} = 16 \cdot 3^y\,, \tag{3}$$

Now, if use (2) and then set

$$z \equiv 3^y\,, \tag{4}$$

we get

$$z^2 + 16z - 4 = 0\,. \tag{5}$$

This has approximate solutions:

$$z_+ = 0.246\,, \quad z_- = -16.246\,. \tag{6}$$

Then, using these values in (4), we get

$$y_+ = \frac{\ln\left[16.246 e^{i\pi} e^{2\pi i n}\right]}{\ln 3} = \frac{\ln\left[16.246\right] + (i\pi + 2\pi i n)}{\ln 3} \quad n \in \mathbb{Z}\,, \tag{7}$$

$$y_- = \frac{\ln\left[0.246 e^{2\pi i n}\right]}{\ln 3} = \frac{\ln\left[0.246\right] + 2\pi i n}{\ln 3} \quad n \in \mathbb{Z}\,. \tag{8}$$

To solve for the corresponding x values, use the relation

$$3^{x_\pm} = \frac{4}{3^{y_\pm}} = \frac{4 e^{2\pi i n}}{z_\pm}\,, \tag{9}$$

and then

$$x_\pm = \frac{\ln\left(\frac{4}{z_\pm}\right) + 2\pi i n}{\ln 3}\,. \tag{10}$$