Math Diversion Problem 654

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> Ramanujan once remarked to Hardy that the number 1729
> is not a dull number, saying, "...it is a very interesting
> number; it is the smallest number expressible as
> the sum of two cubes in two different ways.
> — G.H. Hardy referring to a discussion he
> had with Ramanujan

The YouTube video is found at:

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   Source: https://www.youtube.com/watch?v=Mt1mZKDgKv8
   Title:
A Homemade Equation | Problem 475
    Presenter: aplusbi
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1 The Problem

Given the relation

$$z^i = e^{\frac{\pi}{2}}\,, \tag{1}$$

find the values for z.

2 The Solution

The Given relation can be rewritten as

$$z^i = e^{\frac{\pi}{2}} e^{2\pi i n}\,, \quad n \in \mathbb{Z}\,. \tag{2}$$

Now, raise both sides to the $-i$ power:

$$z = e^{-i\frac{\pi}{2}} e^{2\pi n}\,, \quad n \in \mathbb{Z}\,. \tag{3}$$

Next, we simplify,

$$z = -i e^{2\pi n}\,, \quad n \in \mathbb{Z}\,. \tag{4}$$