Math Diversion Problem 661

P. Reany

June 15, 2025

> I feel that a visual representation of the Dirac algebra is of great
> benefit, because it can provide an additional insight that
> is not easily expressed with words or equations.
> — David M. Goodmanson
> ['A graphical representation of the Dirac algebra',
> American J. Phys., Vol. 64, No. 7,
> July 1996, p. 870.]

The YouTube video is found at:

```
Source: https://www.youtube.com/watch?v=pMGz4fVXqGE
Title: Are YOU smart enough to get into Cambridge?
 Presenter: Math Queen
```

1 The Problem

Given the relation

$$3^x - (\sqrt{3})^{x+4} + 20 = 0\,, \tag{1}$$

find the real values for x, and then add them together.

2 The Solution

The Given relation can be rewritten as

$$(\sqrt{3}^x)^2 - 9(\sqrt{3}^x) + 20 = 0\,, \tag{2}$$

which is quadratic in variable $\sqrt{3}^x$. So, we can apply the quadratic formula.

$$\sqrt{3}^x = \frac{9 \pm \sqrt{81 - 4(1)(20)}}{2} = \frac{9 \pm 1}{2} = 5, 4\,. \tag{3}$$

On taking the natural logarithm across this equation, we have that

$$x\,\tfrac{1}{2}\ln 3 = \ln 5,\ \ln 4\,. \tag{4}$$

or

$$x_1 = 2\frac{\ln 5}{\ln 3}\,, \tag{5}$$

$$x_2 = 2\frac{\ln 4}{\ln 3}\,. \tag{6}$$

Adding these together, we get

$$x_1 + x_2 = 2\left(\frac{\ln 5}{\ln 3} + \frac{\ln 4}{\ln 3}\right) = 2\left(\frac{\ln(5\cdot 4)}{\ln 3}\right) = 2\left(\frac{\ln 20}{\ln 3}\right)\,. \tag{7}$$

However, we can rewrite this result in base 3, to get

$$x_1 + x_2 = 2\left(\frac{\log_3 20}{\log_3 3}\right) = 2\log_3 20\,. \tag{8}$$