Math Diversion Problem 663

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This is a push business, Williams! A push, push, push business!
Push and drive! ... A push, push, push business, Williams!
It's push, push, push — all the way!
— A scolding from the boss to Williams.
The original Twilight Zone.
(I think that those who would excel at mathematics
must adopt this mentality for themselves,
however, it didn't go well for Williams.)

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=1t5EmpnnDe4
Title: Harvard University Admission Interview Tricks
 Presenter: Super Academy
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1 The Problem

Given the relation

$$9^{4^m} = 4^{9^m}\,, \tag{1}$$

find the real values for m.

2 The Solution

Taking the square root across the Given relation, we get

$$3^{4^m} = 2^{9^m}\,. \tag{2}$$

On taking the natural logarithm across this equation, we have that

$$4^m \ln 3 = 9^m \ln 2\,. \tag{3}$$

This can be rewritten as

$$\left(\frac{3}{2}\right)^{2m} = \beta\,, \tag{4}$$

where $\beta \equiv \ln 3/\ln 2$. On taking the natural logarithm across this equation, we have that

$$2m = \frac{\ln \beta}{\ln (3/2)} \,. \tag{5}$$

Thus,

$$m = \frac{\ln (\ln 3/\ln 2)}{2 \ln (3/2)} \,. \tag{6}$$

Alternatively,

$$m = \frac{-\ln (\ln 3/\ln 2)}{-2 \ln (3/2)} = \frac{\ln (\ln 3/\ln 2)^{-1}}{2 \ln (3/2)^{-1}} = \frac{\ln (\ln 2/\ln 3)}{2 \ln (2/3)} \,. \tag{7}$$