Math Diversion Problem 671

P. Reany

June 20, 2025

> The greatest killer of creativity is interruption.
> — John Cleese

The problem is found at:

```
Source: The Ether of Great Mathematical Ideas
Title: A Mixed-rate problem
 Presenter: Patrick
```

1 The Problem

The main oil pump at an oil refinery can fill a tanker in 2 hours. The engineer in charge is concerned that the main pump is in need of repair and he doesn't want to stress it too much. He has a pumping window of 4 hours to fill a tanker before the main pump goes off-line. He also has available a slower pump that can fill the tanker in 6 hours. If the engineer wants to start the tanker job with the slower pump and then add in the main pump at the last possible moment, and then run both simultaneously until the tanker is full, how long will the main pump be used?

2 Solution Part 4.2.1: Conceptualizing the problem

Let's refer to the faster pump (the main pump) as Pump 1, and the other pump as Pump 2. We have 1 job to be accomplish, split into two contributions:

$$1\ [\text{job}] = (\text{PJDB Pump 1}) + (\text{PJDB Pump 2})\,. \tag{1}$$

And the next refinement gives us (suppressing units)

$$1 = R_1 T_1 + R_2 T_2\,, \tag{2}$$

where T_1 is the time the main pump will run, and T_2 is the time the slower pump will run, which we know will be the full 4 hours. Also, we know that $R_1 = \frac{1}{2}[\text{job/hour}]$ is the rate at which the main pump will run, and $R_2 =$

$\frac{1}{6}$[job/hour] is the rate at which the slower pump will operate. Substituting these values into the last equation, we get that

$$1 = \frac{1}{2}\,T_1 + \frac{1}{6}\,4\,. \tag{3}$$

3 Solution Part 4.2.2: Solving the problem

Solving this for T_1 we get,

$$T_1 = \frac{2}{3}\,\text{hour} = 40\ \text{minutes}\,. \tag{4}$$

This means that the engineer needs to turn on the main pump after the slower pump has been running for 3 hours and 20 minutes.

4 Conclusion

Should high-schoolers still learn algebra? Absolutely! Pretend that our recent high school graduate has secured a job as an assistant to the company engineer. Then this unforeseen problem came up. The engineer suddenly became sick and had to leave right away, leaving this simple problem for his assistant to solve. (Which amounts to just pushing the right buttons at the right time.) If she can do it correctly, she can keep her job; otherwise, she'll be fired for being incompetent.

You can't just hand over math problems to machines and expect them to get the right answers all the time. You have to at least know how to solve the problem yourself to double check the result. If you refuse to learn what you need to learn at the right time of your lives, you end up only disadvantaging your older self. Then who's to blame?