Math Diversion Problem 672

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> In mathematics, the art of asking questions is more valuable than solving problems.
> — Georg Cantor

The problem is found at:

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Source: The Ether of Great Mathematical Ideas
Title: A common integral
 Presenter: Patrick
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1 The Problem

$$I = \int x \ln x \, dx \,, \tag{1}$$

We start with the ('Carrington') differential equation:

$$D_x[x^2 \ln x] = 2x \ln x + x^2 \frac{1}{x} = \ln x + x \,. \tag{2}$$

Now we integrate:

$$\begin{aligned} x^2 \ln x &= 2I + \int x \, dx \\ &= 2I + \frac{1}{2}x^2 - C' \,. \end{aligned} \tag{3}$$

From this we get that

$$2I = x^2 \ln x - \frac{1}{2}x^2 + C' \,, \tag{4}$$

or

$$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C \,. \tag{5}$$