

Math Diversion Problem 677

P. Reany

June 24, 2025

It's not that I'm so smart; it's just that
I stay with problems longer.
— Albert Einstein

The problem is found at:

Source: The Ether of Great Mathematical Ideas
Title: The Mystery of Negative Numbers
Presenter: Patrick

1 The Problem

Show that in a ring R containing the integers that

$$(-1)(-1) = 1. \tag{1}$$

2 Solution

In ring R , every element has an additive inverse, so if $a \in R$, then

$$a + (-a) = 0. \tag{2}$$

For all $b \in R$

$$b \cdot 0 = 0. \tag{3}$$

So, let $b = -1$ and replace the zero on the LHS of this last equation with $1 + (-1)$, to get

$$(-1) \cdot (1 + (-1)) = 0. \tag{4}$$

Now, distribute the multiplication by (-1) :

$$(-1)(1) + (-1)(-1) = 0, \tag{5}$$

which simplifies to

$$(-1) + (-1)(-1) = 0. \tag{6}$$

And finally, add unity to both sides.

$$(-1)(-1) = 1. \tag{7}$$