Math Diversion Problem 682

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Dana Scully : The answers are there. You just have to know where to look.
Fox Mulder : That's why they put the 'I' in the FBI.
— The original X-Files

The YouTube video is found at:

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Source: https://www.youtube.com/watch?v=8jhQ5gG_NsY
Title: A Nice Math Olympiad Exponential Equation
 Presenter: MrMath
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1 The Problem

Given the relation

$$x^{x^{16}} = 2^{2^2}\,, \tag{1}$$

find the real values of x.

2 The Solution

Let's begin with an alpha substitution:

$$x = 2^{\alpha}\,, \tag{2}$$

then the Given relation becomes

$$2^{\alpha 2^{16\alpha}} = 2^{2^2}\,. \tag{3}$$

On equating exponents we get

$$\alpha 2^{16\alpha} = 2^2\,. \tag{4}$$

Let's try some easy rational values for α. For starters, $\alpha = 1/2$ won't do it, but $\alpha = 1/4$ works. Hence

$$x = 2^{1/4} \quad \text{or} \quad x = -2^{1/4}\,. \tag{5}$$