

Math Diversion Problem 683

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Ramanujan once remarked to Hardy that the number 1729 is not a dull number, saying, "...it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways.
— G.H. Hardy referring to a discussion he had with Ramanujan
(People often overlook the significance of coincidence.)

The problem is found at:

Source: https://www.algebra.com/algebra/homework/word/travel/Travel_Word_Problems.faq.question.261460.html
Title: Mixed-Rate Problem
Presenter: Patrick

1 The Problem

Question 261460¹: Working together, Sara and Heidi can milk the cows in 2 hours. Working alone, Heidi takes 3 hours longer than Sara (working alone). How long will it take Heidi to milk the cows alone?

2 Solution

We begin with the now standard first level, describing when Sara and Heidi work together:

$$1 \text{ job} = (\text{part of job done by Heidi}) + (\text{part of job done by Sara}). \quad (1)$$

For our next refinement, we use the average rate R [job/hour] at which each woman completes the job of milking all the cows.

$$1 \text{ [job]} = R_H T_H + R_S T_S. \quad (2)$$

¹Found on algebra.com mixture problems

Since we are given that when they work together, they take 2 hours, then we can write that $T_S = T_H = 2$, and we get (suppressing units)

$$1 = R_H T_H + R_S T_S = (R_H + R_S)2. \quad (3)$$

So, how do we relate this equation to what we've been asked to find? Well, what have we been asked to find? We've been asked to find how long it will take Heidi to milk the cows by herself. The time for her to do that is just R_H^{-1} , which has units of hours/job. We require one more relationship to obtain a second equation: We need to convert to an equation the sentence: Working alone, Heidi takes 3 hours longer than Sara (working alone). Therefore, put as an equation, we get

$$R_H^{-1} = R_S^{-1} + 3, \quad (4)$$

or, put into words, the hours/job for Heidi = (the hours/job for Sara) + (3 hours/job).

3 Solution Cont'd

Let's make a variable substitution to simplify the equations: $x = R_H^{-1}$ and $y = R_S^{-1}$. Then (3) and (4) become

$$2x^{-1} + 2y^{-1} = 1 \quad \text{and} \quad x = y + 3. \quad (5)$$

Eliminating y between these equations, yields (after quite some algebra) the quadratic equation

$$x^2 - 7x + 6 = 0, \quad (6)$$

with possible solutions $x = 6, 1$ [hours/job], the latter value being inconsistent with the given information. So, Heidi works 6 hours to complete the job alone.

4 Afterwords:

Sorry, but I often omit the tedious algebra, though I did it myself.