# Math Diversion Problem 689

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June 30, 2025

> Optimism is the faith that leads to achievement. Nothing can be done without hope and confidence.
>
> — Helen Keller

The problem is found at:

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Source: https://www.youtube.com/watch?v=hqux073_Ytc
Title: Harvard Entrance Exam Question
 Presenter:  Math Beast
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## 1 Problem

Given the relation

$$\log_4(\log_3 x)^3 = 4.5\,, \tag{1}$$

find the real values of $x$ that solve this relation.

## 2 Solution

Using a logarithmic identity, we have that

$$3\log_4(\log_3 x) = 4.5\,, \tag{2}$$

which gives us

$$\log_4(\log_3 x) = 1.5 = 3/2\,. \tag{3}$$

On raising 4 to this last equation, we get

$$\log_3 x = 4^{3/2} = 2^3 = 8\,. \tag{4}$$

On raising 3 to this last equation, we get

$$x = 3^8 = 6,561\,. \tag{5}$$