Math Diversion Problem 690

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> Mathematics is the art of reducing any
> problem to linear algebra.
> — William Stein

The problem is found at:

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Source: The Ether of Great Mathematical Ideas
Title:  Mixed-Rate Problem
 Presenter: Patrick
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1 The Problem

John can do a work in 24 days and Ethan can do the same work in 40 days. This time, John starts alone and works for 12 days before Ethan starts and then John stops. Ethan continues the job until the work is 75% finished. How much time T did Ethan work?

2 Solution

Let's try to solve this without the aid of a figure. But we do start with the familiar equation for the job being done by two 'machines' cooperating on the project.

$$.75(\text{job}) = (\text{PJDB John}) + (\text{PJDB Ethan})\,, \tag{1}$$

where, as before, 'PJBD' means 'the part of the job done by'. Then,

$$.75(\text{job}) = (\frac{1}{24}\ \text{job/day})(12\ \text{days}) + (\frac{1}{40}\ \text{job/day})(T)\,. \tag{2}$$

To solve this equation for T, convert .75 to the fraction 3/4, and T is easily found to be 10 days.