

# Math Diversion Problem 691

P. Reany

July 1, 2025

The so-called good among us seem all too eager to  
accept evil if they perceive that they can  
somehow benefit from it.

— The Author

The problem is found at:

Source: <https://www.youtube.com/watch?v=pOH5ex0nsSU>

Title: When Imaginary Meets Euler | Problem 554

Presenter: aplusbi

## 1 Problem

Given the relation

$$i^z = e, \quad (1)$$

find the complex values of  $z$  that satisfy this relation.

## 2 Solution

First, we rewrite the Given to

$$i^z = e^{1+2\pi in} \quad n \in \mathbb{Z}. \quad (2)$$

Next, we take the natural logarithm:

$$z \ln i = z \ln e^{i\pi/2} = 1 + 2\pi in, \quad n \in \mathbb{Z}, \quad (3)$$

which gives us

$$z(i\pi/2) = 1 + 2\pi in, \quad n \in \mathbb{Z}. \quad (4)$$

On dividing through by  $i\pi/2$ , we get

$$z = 4n - \frac{2i}{\pi}, \quad n \in \mathbb{Z}. \quad (5)$$