

Math Diversion Problem 693

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Every big idea needs someone to defend it.
— Cybersecurity

The problem is found at:

Source: <https://www.youtube.com/watch?v=ddwXqK24PGA>

Title: Integral of $(5x^3)/(x^2 + 1)$

Presenter: Calculus Booster

1 Problem

Find the indefinite integral

$$I(x) = \int \frac{x^3}{x^2 + 1} dx. \quad (1)$$

Note: I'm ignoring the factor of 5.

2 Solution

I intend to use a change of variables to a hyperbolic variable. Let

$$x = \sinh y, \quad dx = \cosh y dy. \quad (2)$$

Now,

$$x^2 + 1 = \sinh^2 y + 1 = \cosh^2 y. \quad (3)$$

Thus,

$$\begin{aligned} I(y) &= \int \frac{\sinh^3 y}{\cosh^2 y} \cosh y dy = \int \frac{(\sinh y)(\cosh^2 - 1)}{\cosh y} dy \\ &= \int \sinh y \cosh y dy - \int \tanh y dy \\ &= \frac{1}{2} \sinh^2 y - \ln |\cosh y| + C \end{aligned} \quad (4)$$

So,

$$I(x) = \frac{1}{2}x^2 - \frac{1}{2}\ln(x^2 + 1) + C. \quad (5)$$