Math Diversion Problem 694

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Dear Algebra, stop asking us to find your X,
she's not coming back.
— Woody Paige

The problem is found at:

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Source: https://www.youtube.com/watch?v=mLDj9TJJ7wQ
Title: Harvard University Admission Interview Tricks
 Presenter:  Super Academy
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1 Problem

Given the relation

$$\log_{81} x + \log_9 x = 6\,, \tag{1}$$

solve for x.

2 Solution

I intend to use an alpha substitution: Let

$$x = 81^{\alpha} = 9^{2\alpha}\,. \tag{2}$$

Then, (1) becomes

$$\alpha + \log_9 9^{2\alpha} = 6\,, \tag{3}$$

or,

$$\alpha + 2\alpha = 6\,. \tag{4}$$

Hence,

$$3\alpha = 6\,. \tag{5}$$

So, $\alpha = 2$, making

$$x = 81^2 = 6,561\,. \tag{6}$$