

Math Diversion Problem 705

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Young men should prove theorems, old
men should write books.
— G. H. Hardy

The problem is found at:

Source: <https://www.youtube.com/shorts/hj01mUyNaf8>
Title: Complicated SAT math, linear algebra
Presenter: gotutormaths

1 The Problem

Given the following system of equations

$$3x - ay = 1, \tag{1}$$

$$9x + 6y = b, \tag{2}$$

find the values for a, b such that the system has an infinite solution set, and then determine the value of $b - a$ among the following choices:

A. 5 B. 1 C. 7 D. 2.

2 The Solution

We're given the equations for two lines in a plane. There are only three possibilities: 1) The two lines are non-parallel and thus intersect in a single point. 2) The two lines are parallel and distinct, making no solutions at all. 3) The two lines are the same line, and thus they produce an infinite number of solutions. So, obviously the last case is what we're looking for. But what does that imply.

For two coplanar lines to be the same line, they must be nonzero scalar multiples of each other. So, if we multiply (1) by 3, we will change 3 into 9 (for the coefficients of the x terms between (1) and (2)), and that implies that we have to multiply $-a$ by 3 and also 1 by 3. So, let's get started!

$$3(-a) = 6, \tag{3}$$

and

$$3(1) = b. \tag{4}$$

And this gives us

$$a = -2, \quad b = 3. \tag{5}$$

And finally,

$$b - a = 3 - (-2) = 5. \tag{6}$$