Math Diversion 713

P. Reany

July 11, 2025

> Even the youths shall faint and be weary, and the young men
> shall utterly fall: But they that wait upon the Lord shall
> renew their strength; they shall mount up with wings
> as eagles; they shall run, and not be weary;
> and they shall walk, and not faint.
> — Isaiah 40:30–31

The problem is found at:

```
Source: https://www.youtube.com/watch?v=hVCccahm6WA
Title: logarithm with base and negative argument
 Presenter: Shortredematematica
```

1 The Problem

Given the relation

$$x = \log_{-3}(-27)\,, \tag{1}$$

find the value for x more simply.

2 The Solution

The trick here is to replace $-1 = e^{i\pi}$. Then, using logarithm identities, (1) becomes

$$\begin{aligned} x = \log_{-3}(-27) &= \frac{\log_e(-27)}{\log_e(-3)} \\ &= \frac{\ln(27e^{i\pi})}{\ln(3e^{i\pi})} = \frac{\ln(27) + \ln(e^{i\pi})}{\ln(3) + \ln(e^{i\pi})} \\ &= \frac{\ln(27) + i\pi}{\ln(3) + i\pi}\,. \end{aligned} \tag{2}$$