Math Diversion 724

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July 16, 2025

> This is the normal result of any idea you have in mathematics: It turns out they're either wrong or trivial or a known result.
> — Richard Bocherds

The problem is found at:

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Source: https://www.youtube.com/watch?v=l12TshxBXYE
Title: Can solve this Olympiad Question ?
 Presenter: VIJAY Maths
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The problem is found at:

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Source: https://www.algebra.com/algebra
Title: Question 22466
 Presenter: Patrick
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1 Problem

Given the relation

$$x^3 + \frac{1}{x^3} = 18\,, \tag{1}$$

find the real values of

$$\phi = x^7 + \frac{1}{x^7}\,. \tag{2}$$

2 Solution

My solution plan is simple: Solve for x^3 and substitute that into

$$\phi = (x^3)^{7/3} + \frac{1}{(x^3)^{7/3}}\,. \tag{3}$$

Now, (1) can be rewritten as

$$(x^3)^2 - 18(x^3) + 1 = 0\,, \tag{4}$$

which has positive root

$$x^3 = 9 + \sqrt{80} \approx 17.9443\,. \tag{5}$$

So

$$x^7 \approx 842.9\,. \tag{6}$$

Therefore,

$$\phi \approx 842.9 + \frac{1}{842.9} \approx 842.9\,. \tag{7}$$