Math Diversion 728

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Chop your own wood and it will warm you twice.
— Henry Ford

The problem is found at:

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Source: https://www.youtube.com/watch?v
=Dun-cR6c-1s
Title: Solving a 'Harvard' University entrance exam
 Presenter: Math Olympiad
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1 Problem

Given the relation

$$\sqrt{t} + t = t\sqrt{t}\,, \tag{1}$$

find the real values of t over the positive real numbers.

2 Solution

Let's begin by making a variable change. Let $t \to r^2$, then (1) becomes

$$r + r^2 = r^3\,, \tag{2}$$

which can be rewritten as

$$r(r^2 - r - 1) = 0\,. \tag{3}$$

This has the obvious root $r = 0$, making a root for t as $t = 0$. The other factor gives us

$$r^2 - r - 1 = 0\,, \tag{4}$$

which has roots

$$r = \frac{1}{2}(1 \pm \sqrt{5})\,, \tag{5}$$

thus

$$t = \frac{1}{2}(3 + \sqrt{5})\,, \tag{6}$$

where the negative root was dropped because of the original constraint.