Math Diversion 729

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July 19, 2025

We must use time as a tool, not as a couch.
— John F. Kennedy

The problem is found at:

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Source: The Ether of Great Mathematical Ideas
Title:Mixed Rate
 Presenter: Patrick
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1 Problem

Two vessels A and B have mixtures of milk and water. A has them in ratio $5:2$ and B has them in ratio $8:7$. The volume of vessel A is 2 gallons, and the volume of vessel B is 3 gallons. If the contents of A and B are mixed together, what will be the milk-to-water ratio of this mixture?

2 Solution

We need to find two positive integers x, y to present the final ratio as $x : y$. The problem is that we have only one remaining independent equation we can get out of Figure 1, which is setup to balance for milk. In the process of balancing, we end up with the faction $\dfrac{x}{x+y}$. The question is: How do we use that to get $x : y$? We'll see.

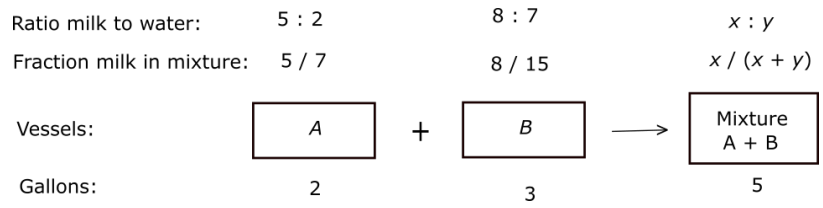

Figure 1. We're optimistically expecting to find two relatively prime positive integers x, y such that we can write the final answer as the ratio $x : y$.

3 Solution Part 11.4.2: Solving the problem

From the conservation of milk, we get that

$$\frac{5}{7} \cdot 2 + \frac{8}{15} \cdot 3 = \frac{x}{x+y} \cdot 5\,. \tag{1}$$

Let's now employ a trick we've used before. Let $\lambda = x/y$ and rewrite $\dfrac{x}{x+y}$ as $\dfrac{\lambda}{\lambda+1}$. Now, we can rewrite (1) as

$$(\frac{10}{7} + \frac{24}{15})(\lambda + 1) = 5\lambda\,. \tag{2}$$

Wolframalpha.com gives the solution to λ as $\dfrac{106}{69}$. Therefore, $x : y :: 106 : 69$.