Math Diversion 731

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I love it when a plan comes together.
— Hannibal Smith, *The A-Team*

The problem is found at:

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Source: https://math.stanford.edu/~vakil
      /putnam02/02/putnam3.pdf
Title:  COMPLEX NUMBERS
 Presenter:  ALOK AGGARWAL and RAVI VAKIL
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1 Problem A5

Suppose $\cos\theta = 1/\pi$. Evaluate:

$$\phi = \sum_{n=0}^{\infty} \frac{\cos\, n\theta}{2^n}\,. \tag{1}$$

2 Solution

ϕ can be rewritten as

$$\phi = \sum_{n=0}^{\infty} \left(\frac{\cos\,\theta}{2}\right)^n = \sum_{n=0}^{\infty} \left(\frac{1}{2\pi}\right)^n\,. \tag{2}$$

Lemma: The geometric series. Assume $|\,r\,| < 1$, then

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}\,. \tag{3}$$

Thus, with $r = \dfrac{1}{2\pi} < 1$, we can apply the above formula:

$$\phi = \frac{1}{1-(1/2\pi)} = \frac{2\pi}{2\pi-1}\,. \tag{4}$$