Math Diversion 733

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> Unless you try to do something beyond what you have
> already mastered, you will never grow.
> — Ralph Waldo Emerson

The problem is found at:

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Source: https://www.youtube.com/watch?v=gWj-gXjCceg
Title:  Can You Solve This Advanced Logarithm Equation?
 Presenter:  Khem math
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1 Problem

Actually, the problem I will solve is a slight adaptation of the one the Presenter gave.

Given the relation

$$x^{2+\ln x} = 0.1\,, \tag{1}$$

solve for x.

2 Solution

First, let's rewrite the Given as

$$x^{2+\ln x} = 10^{-1}\,, \tag{2}$$

and then take the logarithm base 10 across, to get

$$(2+\ln x)\log x = -1\,, \tag{3}$$

But

$$\ln x = \frac{\log x}{\log e} \equiv \alpha \log x\,, \tag{4}$$

where

$$\alpha \equiv \frac{1}{\log e} \approx 2.303\,. \tag{5}$$

Therefore, (5) becomes

$$\alpha(\log x)^2 + 2\log x + 1 = 0\,. \tag{6}$$

On setting $y = \log x$, we get

$$\alpha y^2 + 2y + 1 = 0\,. \tag{7}$$

The roots to this are

$$y = -0.434 \pm i0.495\,. \tag{8}$$

Hence,

$$\log x = -0.434 \pm i0.495\,. \tag{9}$$

On raising 10 to this equation, we have that

$$x = 10^{-0.434 \pm i0.495} = 10^{-0.434}10^{\pm i0.495} = 0.368 \times 10^{\pm i0.495}\,. \tag{10}$$

Lastly, we get (approximately)

$$x = 0.154 \pm 0.335i\,. \tag{11}$$