Math Diversion 738

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Theory like mist on eyeglasses — obscures facts.
— Charlie Chan

The problem is found at:

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Source: https://www.youtube.com/watch?v=VZT_0kgGbF8
Title:  Can You Crack This Power Puzzle?
 Presenter:  SyberMath
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1 Problem

Given the relation

$$5^x = 4\,, \tag{1}$$

find the real value of

$$\phi = 20^{\frac{x}{x+1}}\,. \tag{2}$$

2 Solution

One way to proceed would be to solve (1) for x and then to use that value in (2). But there is another way.

We begin by multiplying (1) through by 4^x to get

$$4^x \cdot 5^x = 4^{x+1}\,. \tag{3}$$

But we can rewrite this as

$$20^x = 4^{x+1}\,. \tag{4}$$

Next, we take $(x+1)$st root to both sides, to get

$$20^{\frac{x}{x+1}} = 4\,. \tag{5}$$

Hence,

$$\phi = 4\,. \tag{6}$$