Math Diversion Problem 741

P. Reany

July 26, 2025

> Self-education is, I firmly believe, the only
> kind of education there is.
> — Isaac Asimov

The material here is found at:

```
Source: The Ether of Great Mathematical Ideas
Title: The entropy of a reversible process on an ideal gas
 Presenter: Patrick
```

1 The Problem

Show that the entropy of a reversible process on an ideal gas is unchanged over a single cycle.

2 The Setup

We will model the process in a P-V diagram. We'll be interested in accounting for the change in entropy S over a single cycle.

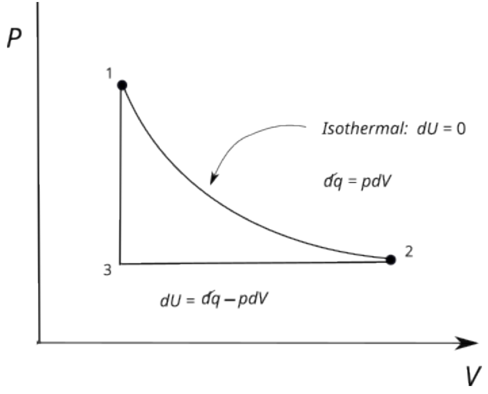

Figure 1. For the path 1 to 2, the ideal gas undergoes isothermal expansion. For 2 to 3, the gas undergoes compression at constant pressure. From 3 to 1, the gas undergoes pressurization at constant volume (hence no work being done).

By the way, the relation governing an ideal gas is:

$$PV = nRT\,. \tag{1}$$

3 The Solution

Every total is equal to the sum of its parts: Same for entropy.

$$\sum \Delta S = \Delta S_{1\to 2} + \Delta S_{2\to 3} + \Delta S_{3\to 1}\,. \tag{2}$$

$$\Delta S_{1\to 2} = \int_1^2 \frac{\text{đ}q}{T} = \int_1^2 \frac{p\,dV}{T} = nR\int_1^2 \frac{dV}{V} = nR\ln\frac{V_2}{V_1}\,. \tag{3}$$

►

$$\begin{aligned}\Delta S_{2\to 3} &= \int_2^3 \frac{\text{đ}q}{T} = \int_2^3 \frac{nR\,dT}{T} + \int_2^3 \frac{P}{T}dV \\ &= nR\ln\frac{T_3}{T_2} + nR\ln\frac{V_3}{V_2}\,. \end{aligned} \tag{4}$$

►

$$\Delta S_{3\to 1} = \int_3^1 \frac{nR\,dT}{T} = nR\ln\frac{T_3}{T_2}\,. \tag{5}$$

►

$$\sum \Delta S = nR\left[\ln\frac{T_3}{T_2} + \ln\frac{T_1}{T_3} + \ln\frac{V_3}{V_2} + \ln\frac{V_2}{V_1}\right] \quad = nR\left[\ln\frac{T_1}{T_2} + \ln\frac{V_3}{V_1}\right]\,. \tag{6}$$

But points 1 and 2 are connected by an isotherm, hence $T_1 = T_2$, hence

$$\ln\frac{T_1}{T_2} = 0\,. \tag{7}$$

Likewise $V_3 = V_1$ because points 1 and 3 are connected by a line of constant volume. Hence

$$\ln\frac{V_3}{V_1} = 0\,, \tag{8}$$

implying that the total change in entropy around a single loop of the process is zero.