# Math Diversion 744

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No mystery is closed to an open mind.
— Tim White, host of *Sightings*

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Source: The Ether of Great Mathematical Ideas
Title: Specific Heats of the Ideal Gas Relation
 Presenter:  Patrick
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## 1 Preliminaries

The specific heat for an ideal gas at a constant volume is defined as

$$c_V \equiv \left(\frac{\partial U}{\partial T}\right)_V , \tag{1}$$

where $U$ is the internal energy of the gas. Similarly, the specific heat for an ideal gas at a constant pressure is defined as

$$c_P \equiv \left(\frac{\partial H}{\partial T}\right)_P , \tag{2}$$

where $H$ is the enthalpy of the gas.

The fundamental Ideal Gas Law is

$$PV = nRT \, , \tag{3}$$

where $n$ is the number of moles of the gas, and $R$ is the Ideal Gas Constant.

The enthalpy $H$ is related to $U, P, V$ by

$$H = U + PV \, , \tag{4}$$

which is generally true for all substances.

And now we have one last sneaky relation to sneak in. For an ideal gas, the internal energy $U$ is only a function of temperature $T$, or

$$U = U(T) \, . \tag{5}$$

And if you are presently studying thermodynamics, keep this relation in mind whenever the substance is an ideal gas.

## 2 The Problem

Show that

$$\bar{c}_P = \bar{c}_V + R\,, \tag{6}$$

where $\bar{c}_P$ and $\bar{c}_V$ are the molar ideal gas specific heats at constant pressure and constant volume, respectively.

## 3 The Solution

Combining (4) and (3), we get

$$H = U + nRT\,. \tag{7}$$

On differentiating this last equation by $T$, holding $P$ constant, we have that

$$\left(\frac{\partial H}{\partial T}\right)_P = \left(\frac{\partial U}{\partial T}\right)_P + nR\,. \tag{8}$$

Using (2), we get

$$c_P = \left(\frac{\partial U}{\partial T}\right)_P + nR\,. \tag{9}$$

Now, because of (5),

$$\left(\frac{\partial U}{\partial T}\right)_P = \left(\frac{\partial U}{\partial T}\right)_V\,. \tag{10}$$

So, using this last equation and (1), we have that

$$c_P = c_V + nR\,. \tag{11}$$

On dividing through by the number of moles $n$, we get the molar equivalent of this equation,

$$\bar{c}_P = \bar{c}_V + R\,, \tag{12}$$

which is what we were to show.