

# Math Diversion 745

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You must do what you're  
called to do in this life.  
— Hetty Wainthropp

Source: <https://www.youtube.com/watch?v=myn4KzdA-CU>  
Title: Nice Math Olympiad Question  
Presenter: Math Beast

## 1 The Problem

Given the relation

$$\log_2 x + \log_3 x = 1, \quad (1)$$

find the real values of  $x$ .

## 2 The Solution

Let's try to solve this with a variable substitution.

Let

$$x = 2^\alpha, \quad (2)$$

then (1) becomes

$$\alpha + \log_3 2^\alpha = 1. \quad (3)$$

From this we can solve for  $\alpha$ , getting

$$\alpha = \frac{1}{1 + \log_3 2}. \quad (4)$$

Hence

$$x = 2^{(1/(1+\log_3 2))}. \quad (5)$$

If you prefer the logarithms in base 10, then

$$x = 2^{\log 3 / \log 6}. \quad (6)$$