Math Diversion 747

P. Reany

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When you're properly incentivized,
you can deceive yourself.
— Jared Henderson

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Source: https://www.youtube.com/watch?v=gmGLBhS4fXY
Title: A Nice Nonstandard Equation
 Presenter:  SyberMath
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1 The Problem

Given the relation

$$4^x = -x\,, \tag{1}$$

find the real values of x.

2 The Solution

Let's rewrite the Given to

$$1 = -x4^{-x}\,. \tag{2}$$

Then take the Lambert W function across this:[1]

$$-x = W_{(4)}(1) = \frac{W(1\cdot\ln 4)}{\ln 4} = \frac{W(2\ln 2)}{2\ln 2} = \frac{\ln 2}{2\ln 2} = \frac{1}{2}\,. \tag{3}$$

Therefore

$$x = -\frac{1}{2}\,. \tag{4}$$

3 Appendix: Lambert

Sometimes I need to use the Lambert W function, which goes as follows: If

$$ze^z = B\,, \tag{5}$$

[1]See the Appendix for help with the Lambert W function.

then

$$z = W(B)\,, \tag{6}$$

where there are domain constraints on B that we won't go into here. Warning: This can be a complicated (multi-valued) function to deal with.

A lemma I'll need from the theory of the Lambert W function is the following: If

$$y \ln y = B\,, \tag{7}$$

then

$$\ln y = W(y \ln y) = W(B)\,. \tag{8}$$

The following is the 'Lambert W function base s'[2], or W_s, where s is a positive real number. Let's begin with the relation

$$xs^x = A\,, \tag{9}$$

which looks very similar to (7). Then

$$x = W_s(xs^x) \equiv \frac{W(A \ln s)}{\ln s}\,. \tag{10}$$

But when $s = e$, we have that

$$x = W_e(xe^x) = \frac{W(A \ln e)}{\ln e} = W(A)\,, \tag{11}$$

which is the usual Lambert W function. (By the way, the proof to this lemma is not hard. It begins with setting $s^x = e^y$ and proceeding from there.)

If s is an integer, I may resort to putting parentheses around it to distinguish it from the n-series, as such $W_{(s)}$.

One last result we might need is

$$\gamma = W_n(\gamma) e^{W_n(\gamma)}\,. \tag{12}$$

[2]This notation I invented myself.