

# Math Diversion 748

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My grandparents's generation used to say, "It will NEVER happen!" My parents's generation said, "Not in MY lifetime, it won't." Our generation has to get used to saying, "Probably not before Tuesday."

— Anonymous

Source: The Ether of Great Mathematical Ideas

Title: A Nice Mixed-Rate Problem

Presenter: Patrick

## 1 The Problem

$A$  and  $B$  together can do a job in 8 days.  $A$  and  $C$  together can do the job in 9 days. And  $B$  and  $C$  together can do the job in 10 days. What is  $B$ 's individual rate?

## 2 The Solution

When two 'machines',  $A$  and  $B$ , say, work together over a common time they have a combined effective rate of  $\frac{1}{T}$ , which we get from the equation

$$(R_A + R_B)T = 1[\text{job}], \quad (1)$$

from which we get

$$R_{A+B}^{\text{effective}} = R_A + R_B = \frac{1}{T}. \quad (2)$$

Now, on to the rate we need. So, from the given information, we get

$$R_A + R_B = 1/8, \quad (3a)$$

$$R_A + R_C = 1/9, \quad (3b)$$

$$R_B + R_C = 1/10. \quad (3c)$$

According to wolframalpha.com,  $R_B = \frac{41}{720}$ .