# Math Diversion 749

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> More than a century since its debut, representation theory
> has served as a key ingredient in many of the most important
> discoveries in mathematics. Yet its usefulness
> is still hard to perceive at first.
> — Kevin Hartnett

(Representation theory — among other uses, it is the representation of elements of an arbitrary group by the elements of a linear map on a vector space. Once a basis is chosen, the linear map can take the form of a matrix group.)

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Source: The Ether of Great Mathematical Ideas
Title: A Nice Mixed-Rate Problem (Continuation)
 Presenter: Patrick
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## 1 The Problem

In the last problem, we had this to do: $A$ and $B$ together can do a job in 8 days. $A$ and $C$ together can do the job in 9 days. And $B$ and $C$ together can do the job in 10 days. What is $B$'s individual rate?

This time, given the same information, determine how long it will take to have the job completed if all three start and stop on the project till it's finished. However, it's not allowed to compute the individual rates to compute the answer.

## 2 The Solution

When two 'machines', $A$ and $B$, say, work together over a common time they have a combined effective rate of $\frac{1}{T}$, which we get from the equation

$$(R_A + R_B + R_C)\bar{T} = 1[\text{job}]\,, \tag{1}$$

where $\bar{T}$ is the common time they will all work together.

Last time, we wrote down these equations

$$R_A + R_B = 1/8\,, \tag{2a}$$
$$R_A + R_C = 1/9\,, \tag{2b}$$
$$R_B + R_C = 1/10\,. \tag{2c}$$

Look what happens when we just add these three equation together.

$$2(R_A + R_B + R_C) = \frac{1}{8} + \frac{1}{9} + \frac{1}{10} = \frac{242}{360}\,. \tag{3}$$

Thus, From (1)

$$2\overline{T}^{-1} = \frac{242}{720}\,. \tag{4}$$

Hence,

$$\overline{T} = \frac{720}{121}\,. \tag{5}$$