

Math Diversion 751

P. Reany

August 7, 2025

It is said, Honor dies where interest lies.

— Kwai Chang Caine

Source: <https://math.stackexchange.com/questions/147324/min-max-of-fx-y-exy-where-x3y3-16>
Title: A Lagrange multipliers problem
Presenter: Asker

1 The Problem

Given the relation

$$f(x, y) = e^{xy}, \quad (1)$$

subject to the constraint

$$g(x, y) = x^3 + y^3 = 16, \quad (2)$$

find the critical points of this system.

2 The Solution

We will use the method of Lagrange multipliers to solve this problem. Hence, with the understanding that

$$\nabla = \sigma_1 \partial_x + \sigma_2 \partial_y, \quad (3)$$

we write

$$\nabla f(x, y) + \lambda \nabla g(x, y) = 0, \quad (4)$$

where λ is a Lagrange multiplier, which we won't need to solve for. First, we rewrite (4) to this

$$\nabla f(x, y) = -\lambda \nabla g(x, y), \quad (5)$$

then we write

$$(\sigma_1 \partial_x + \sigma_2 \partial_y) e^{xy} = -\lambda (\sigma_1 \partial_x + \sigma_2 \partial_y) (x^3 + y^3). \quad (6)$$

From this we get

$$\sigma_1 y e^{xy} + \sigma_2 x e^{xy} = -\lambda[\sigma_1 3x^2 + \sigma_2 3y^2]. \quad (7)$$

On equating like components, we get

$$y e^{xy} = -\lambda 3x^2, \quad (8a)$$

$$x e^{xy} = -\lambda 3y^2. \quad (8b)$$

By dividing these last two equations, we have that

$$\frac{x}{y} = \frac{3x^2}{3y^2}, \quad (9)$$

which simplifies down to

$$y^3 = x^3, \quad (10)$$

and so

$$y = x, \quad (11)$$

If we put this last equality into (2), we get

$$2x^3 = 16, \quad (12)$$

which gives us $x = 2$. But from (11) we also get $y = 2$. Therefore we have the single critical point $(2, 2)$.