Math Diversion Problem 754

P. Reany

August 11, 2025

A clue is anything that doesn't happen
the way it oughtta happen.
— Harry Orwell, TV
show *Harry O*

The material here is found at:

```
Source: https://www.youtube.com/watch?v=Jsn6-_qTTDw
Title: Tricky Maths Questions for Competitive Exams
 Presenter: Math Beast
```

1 The Problem

Given the relation

$$\frac{e^x - e^{-x}}{3} = 1\,, \tag{1}$$

find the values of x.

2 The Preparation

There are a number of ways to solve this problem. I wish to use a method that embraces hyperbolic trig functions. Useful facts,

$$\frac{e^x - e^{-x}}{2} = \sinh x\,, \tag{2}$$

and for another

$$\sinh^{-1} u = \ln\left[u + \sqrt{u^2 + 1}\,\right]. \tag{3}$$

3 The Solution

Let's begin by rewriting (1) to the form

$$\frac{e^x - e^{-x}}{2} = \frac{3}{2}\,. \tag{4}$$

On using (2), we have that

$$\sinh x = \frac{3}{2}\,, \tag{5}$$

and on using (3), we get

$$x = \sinh^{-1}\frac{3}{2} = \ln\left[\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 + 1}\right]\,. \tag{6}$$

or

$$x = \ln\left(\frac{3+\sqrt{13}}{2}\right). \tag{7}$$