

Math Diversion Problem 757

P. Reany

August 14, 2025

If you want that tool you're about to buy to last
more than three usages, never go cheap.
— The Author

The material here is found at:

Source: <https://www.youtube.com/watch?v=sijD0cX4RFc>
Title: Japanese Exponential Equation
Presenter: Mathpoints

1 The Problem

Given the relation

$$2^{3^x} = 3^{2^x}, \quad (1)$$

find the real values of x .

2 The Solution

Let's begin by taking the logarithm base 2 across (1), to get

$$3^x = 2^x \log_2 3, \quad (2)$$

which then becomes

$$\left(\frac{3}{2}\right)^x = \log_2 3 = \frac{\log 3}{\log 2}. \quad (3)$$

Now we take the logarithm base 10 across this, to get

$$x = \frac{\log(\log 3 / \log 2)}{\log(3/2)} = \frac{\log(\log 3) - \log(\log 2)}{\log 3 - \log 2}. \quad (4)$$