Math Diversion Problem 759

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> Young men should prove theorems, old men should write books.
> — G. H. Hardy

The material here is found at:

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Source: https://www.youtube.com/watch?v=-BsTqhZKeCw
Title: Can you solve?
 Presenter: Mathpoints
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1 The Problem

Given the relation

$$a + b = 5\sqrt{ab}\,, \tag{1}$$

find the real values of a/b.

2 The Solution

Let

$$\eta \equiv \frac{a}{b}\,, \tag{2}$$

then, after dividing (1) through by b, it becomes

$$\eta + 1 = 5\sqrt{\eta}\,, \tag{3}$$

or even better

$$\eta - 5\sqrt{\eta} + 1 = 0\,, \tag{4}$$

which is a quadratic in $\sqrt{\eta}$. Hence,

$$\sqrt{\eta} = \frac{5 \pm \sqrt{21}}{2}\,, \tag{5}$$

with solutions for η:

$$\eta_+ = \frac{23 + 5\sqrt{21}}{2} \quad \text{and} \quad \eta_- = \frac{2}{23 + 5\sqrt{21}}\,, \tag{6}$$