Math Diversion Problem 768

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Dear Algebra, stop asking us to find your X,
she's not coming back.
— Woody Paige

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Source: The Ether of Great Mathematical Ideas
Title:  An Integration Problem
 Presenter: Patrick
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Find the integral

$$I = \int \frac{\tan x}{1 + \tan^2 x}\, dx\,. \tag{1}$$

But, $1 + \tan^2 x = \sec^2 x = \dfrac{1}{\cos^2 x}$ and $\tan x = \dfrac{\sin x}{\cos x}$, so that

$$I = \int \sin x \cos x\, dx\,. \tag{2}$$

At this point, I could resort to using another trig identity, but I'd prefer to do this more simply. Given that $\cos x\, dx = d(\sin x)$, then

$$I = \int \sin x\, d(\sin x)\,, \tag{3}$$

which has the form of

$$\int u\, du = \tfrac{1}{2}u^2 + C\,. \tag{4}$$

Thus,

$$I = \int \sin x\, d(\sin x) = \tfrac{1}{2}\sin^2 x + C\,. \tag{5}$$

However, WolframAlpha claims that the correct answer is

$$I = \int \sin x\, d(\sin x) = -\tfrac{1}{2}\cos^2 x + C'\,, \tag{6}$$

but these are equivalent because of

$$\cos^2 x + \sin^2 x = 1\,. \tag{7}$$