

# Math Diversion Problem 769

P. Reany

August 21, 2025

People often overlook the obvious.  
— Doctor Who

Source: <https://www.youtube.com/watch?v=skIwyzuV2mQ>  
Title: Solving a 'Harvard' University Entrance Exam Question  
Presenter: Maths Explorer

## 1 Problem

Given the relation

$$\log_2 x + \log_5 x = 6, \quad (1)$$

find the real values of  $x$

## 2 Solution

We need to convert to a single base, but which one? If there were an  $e$  in the problem, that might influence me to go with natural logs. Failing that, my default choice is base 10. Thus, (1) becomes

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 5} = 6. \quad (2)$$

On factoring out the  $\log x$ , we get

$$\log x \left( \frac{1}{\log 2} + \frac{1}{\log 5} \right) = 6. \quad (3)$$

Now, since

$$\frac{1}{\log 2} + \frac{1}{\log 5} = \frac{\log 2 + \log 5}{(\log 2)(\log 5)} = \frac{\log 10}{(\log 2)(\log 5)} = \frac{1}{(\log 2)(\log 5)}, \quad (4)$$

then

$$\log x = 6(\log 2)(\log 5). \quad (5)$$

And finally,

$$x = 10^{6(\log 2)(\log 5)}. \quad (6)$$