Math Diversion Problem 772

P. Reany

August 23, 2025

> First things first...But not necessarily in that order.
> — Doctor Who

```
Source: https://www.youtube.com/watch?v=bAGRzVh752o
Title:  Nice Math Olympiad Algebra Question
 Presenter: Math Beast
```

1 Problem

Given the relation

$$t^4 - 4^t = 17\,, \tag{1}$$

find the positive integer solutions of t.

2 Solution

Given that we are tasked only to find integer solutions to (1), we should try to solve it by use of a table.

t	t^4	4^t	$t^4 - 4^t$
2	16	16	0
3	81	64	$81 - 64 = 17$

Table 1: These types of problems rarely require large values.

The result $t = 3$ is the only solution because the two functions $f(t) = t^4$ and $g(t) = 4^t + 17$ are both monotonically increasing functions (for $t > 0$) and their graphs will intersect over the real numbers at most once.