Math Diversion Problem 773

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> Keep an open mind. That's the secret.
> — Doctor Who

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Source: https://www.youtube.com/watch?v=-5qiRIOteU4
Title:  Solve a logarithmic equation. Japanese
 Presenter: Maths Enhancer's Class
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1 Problem

Given the relation

$$\log_5(3^x + 4^x) = \log_4(5^x - 3^x)\,, \tag{1}$$

find a positive integer solution of x. (Note: The original problem is stated more generally.)

2 Solution

Set

$$\log_5(3^x + 4^x) = \log_4(5^x - 3^x) = y\,, \tag{2}$$

and then

$$3^x + 4^x = 5^y\,, \tag{3}$$

$$5^x - 3^x = 4^y\,. \tag{4}$$

Adding these last two equations together, we have that

$$5^x + 4^x = 5^y + 4^y\,. \tag{5}$$

Since x and y are not independent of each other, one obvious relation between them is

$$x = y\,. \tag{6}$$

Using this constraint in (3), we get

$$3^x + 4^x = 5^x\,. \tag{7}$$

We know that a valid solution to this last equation is

$$x = 2\,. \tag{8}$$