Math Diversion Problem 778

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Physical concepts are free creations of the human mind,
and are not, however it may seem, uniquely
determined by the external world.
— Albert Einstein

`Source: https://www.youtube.com/watch?v=caHAahy9W30`
`Title: A Nice Algebra Problem`
`Presenter: SALogic`

1 Problem

Given the relation

$$\frac{2^{x^2}}{4^x} = 6\,, \tag{1}$$

solve for the values of x.

2 Solution

Let's rewrite (1) as

$$\frac{2^{x^2}}{2^{2x}} = 2 \cdot 3\,, \tag{2}$$

and this can be rewritten as

$$2^{x^2-2x-1} = 3 = 3e^{2\pi i n}, \qquad n \in \mathbb{Z}\,. \tag{3}$$

On taking the natural log, we have that

$$(x^2 - 2x - 1)\ln 2 = \ln 3 + 2\pi i n\,. \tag{4}$$

By dividing through by $\ln 2$, we get

$$x^2 - 2x - 1 = (\ln 3 + 2\pi i n)/(\ln 2)\,. \tag{5}$$

From which we get the quadratic in x:

$$x^2 - 2x - \left(\frac{\ln 3 + 2\pi i n}{\ln 2} + 1\right) = 0\,, \tag{6}$$

or rather

$$x^2 - 2x - \left(\frac{\ln 6 + 2\pi i n}{\ln 2}\right) = 0\,. \tag{7}$$

The reader may continue at this point to solve for the general case, but I will only solve for the $n = 0$ cases. So we have that

$$x^2 - 2x - \left(\frac{\ln 6}{\ln 2}\right) = 0\,. \tag{8}$$

Therefore

$$\begin{aligned} x &= \frac{2 \pm \sqrt{4 - 4(1)(-\ln 6/\ln 2)}}{2} \\ &= 1 \pm \sqrt{1 + \ln 6/\ln 2} \\ &= 1 \pm \sqrt{\ln 12/\ln 2} \\ &= 1 \pm \sqrt{\frac{\log 12}{\log 2}}\,. \end{aligned} \tag{9}$$