# Math Diversion Problem 779

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> Equations are just the boring part of mathematics. I attempt to see things in terms of geometry.
> — Stephen Hawking

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Source: https://www.youtube.com/watch?v=C87XehBs0kg
Title: Oxford University  | Entrance Exam
 Presenter: Leo Dorber
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## 1 Problem

Given the relation

$$x^x = 3^{18}\,, \tag{1}$$

solve for the real values of $x$.

## 2 Solution

Let's use an alpha substitution. Let

$$x = 3^\alpha\,. \tag{2}$$

On substituting this into (1), we have that

$$(3^\alpha)^{3^\alpha} = 3^{18}\,, \tag{3}$$

or even better

$$3^{\alpha 3^\alpha} = 3^{18}\,. \tag{4}$$

On equating exponents, we get

$$\alpha 3^\alpha = 18 = 2 \cdot 3^2\,. \tag{5}$$

We can solve this equation by inspection, getting

$$\alpha = 2\,. \tag{6}$$

Hence

$$x = 9\,. \tag{7}$$