Math Diversion Problem 783

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If you're not in love with the Truth, you could be
talked into believing almost anything.
— Author

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Source: https://www.youtube.com/watch?v=8qbB7o-fqqc
Title:  Trig Meets the Imaginary Realm | P 565
 Presenter: aplusbi
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1 Problem

Given the relation

$$\sec\theta + \tan\theta = i\,, \tag{1}$$

solve for θ.

2 Solution

Since $\sec\theta = 1/\cos\theta$, the Given becomes

$$1 + \sin\theta = i\cos\theta\,, \tag{2}$$

with the condition that $\cos\theta \neq 0$, to be consistent with (1). Next, we multiply (2) though by i:

$$i + i\sin\theta = -\cos\theta\,, \tag{3}$$

which can be rearranged to

$$\cos\theta + i\sin\theta = -i = e^{3\pi i/2} = e^{3\pi i/2}e^{2\pi in} \quad \text{where} \quad n \in \mathbb{Z}\,. \tag{4}$$

So, the LHS of this last equation is $e^{i\theta}$ and the RHS is $e^{3\pi i/2+2\pi in}$. Therefore, we have that

$$e^{i\theta} = e^{3\pi i/2+2\pi in} \quad \text{where} \quad n \in \mathbb{Z}\,. \tag{5}$$

On equating exponents and dividing through by i, we get

$$\theta = 3\pi/2 + 2\pi n \quad \text{where} \quad n \in \mathbb{Z}\,. \tag{6}$$