

Math Diversion Problem 788

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Source: <https://www.youtube.com/watch?v=pMGz4fVXqGE>
Title: Are YOU smart enough to get into Cambridge?
Presenter: Math Queen

1 Problem

Given the relation

$$3^x - (\sqrt{3})^{x+4} + 20 = 0, \quad (1)$$

solve for the sum of the real values of x .

2 Solution

Eq. (1) simplifies to

$$3^x - 9(\sqrt{3})^x + 20 = 0, \quad (2)$$

or

$$(3^{x/2})^2 - 9(3^{x/2}) + 20 = 0. \quad (3)$$

Hence,

$$3^{x/2} = \frac{9 \pm \sqrt{81 - 4(1)(20)}}{2} = \frac{9 \pm 1}{2} = 5, 4. \quad (4)$$

So, let's square:

$$3^x = 25, 16. \quad (5)$$

Now let's take the logarithm base 3 to both sides:

$$x = \log_3 25, \log_3 16. \quad (6)$$

To get the sum of these values, we just add them:

$$\begin{aligned} \text{Sum} &= \log_3 25 + \log_3 16 \\ &= 2 \log_3 5 + 2 \log_3 4 \\ &= 2(\log_3 5 + \log_3 4) \\ &= 2(\log_3 5 \cdot 4) = 2 \log_3 20. \end{aligned} \quad (7)$$