Math Diversion Problem 789

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September 7, 2025

Chancellor, all presidents are faced with difficult decisions.
It is by their *decisions* that they are judged.
— Cardinal Barusa
[Doctor Who, *Deadly Assassin*]

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Source: https://www.youtube.com/watch?v=L8DqynoKGUE
Title:  A Complex Exponential Equation | Problem 343
 Presenter: aplusbi
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1 Problem

Given the relation

$$i^{iz} = 1\,, \tag{1}$$

solve for the complex values of z.

2 Solution

First, we perform our standard tricks.

$$i^{iz} = \left(i^i\right)^z = 1 = e^{2\pi in} \quad \text{where} \quad n \in \mathbb{Z}\,. \tag{2}$$

Next,

$$i^i = \left(e^{\pi i/2}\right)^i = e^{-\pi/2}\,, \tag{3}$$

and we did not this time add in another infinite number of complex solutions, because we already did that.

Therefore, (2) becomes

$$i^{iz} = e^{-\pi z/2} = e^{2\pi in} \quad \text{where} \quad n \in \mathbb{Z}\,, \tag{4}$$

On taking the natural logarithm of both sides, we get

$$-\pi z/2 = 2\pi in \quad \text{where} \quad n \in \mathbb{Z}\,. \tag{5}$$

Solving for z, we have that

$$z = -4in \quad \text{where} \quad n \in \mathbb{Z}\,. \tag{6}$$